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SOME EXPERIMENTAL RESULTS ON COMMUNICATION SYSTEMS SUBJECT TO INTERSYMBOL INTERFERENCE

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NOTICE

Preface

If, in a digital communication system, one attempts to communicate at a rate which approaches or exceeds the reciprocal of the channel's correlation bandwidth intersymbol interference results and performance is degraded. This report investigates the degree to which this degradation can be compensated for by the use of easily implemented phase correcting networks.

The results of the study show that a significant improvement in system performance can be achieved by using compensating networks composed of cascaded single pole all-pass sections. For instance, for a delay distortion factor of 1.5 (envelope delay at each end of the band is ± 1.5 baud lengths relative to the midband spectrum component) probability of error increases from .1 to .15 without the use of phase correction networks. A two section compensating network will result in only one-third as much degradation, while a five section compensating network virtually reduces the degradation to zero.

Some Experimental Results on Communication Systems

Subject to Intersymbol Interference

In order to determine some of the effects of channel phase distortion on the probability of error in digital communication systems, an experimental model of a binary PSK system was developed using the digital computer. A block diagram of the system simulated is shown in Figure 1

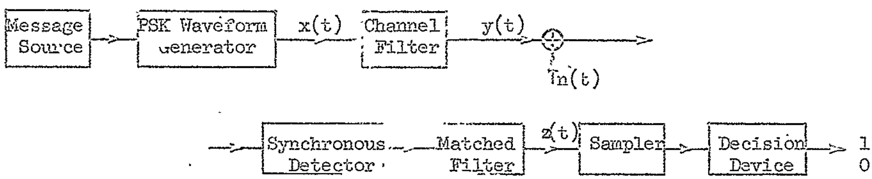

Figure 1

The communication system model employed a raised cosine spectrum for the signaling waveforms, a time invariant channel filter, additive bandlimited white Gaussian noise, phase-coherent syncronous detection and an ideal matched filter detector perfectly syncronized with the received waveform. Results were obtained which indicated the effect of various amounts of linear delay distortion on the probability of error per received digit for fixed signal-to-noise ratio. Results were also obtained indicating the improvement in probability of error which occurs when a number of all-pass networks are used to provide phase correction for the combating of the channel-induced phase distortion.

The computer program was developed according to the block diagram of Figure 2. All the operations in this diagram represent equivalent lowpass operations performed on the signal envelope. The pseudo-random message source modulated the waveform generator with a 127-bit maximum-length binary sequence.

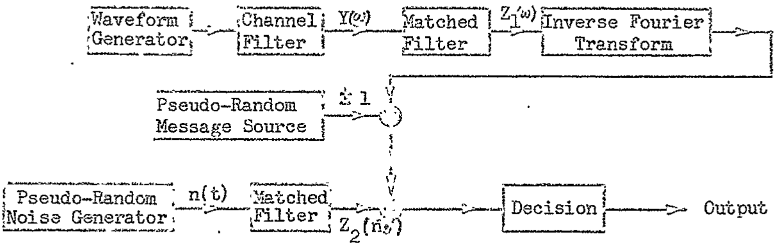

Figure 2.

It can be seen in this diagram that the computations associated with the signal waveform's passage through the linear portions of the system are performed with frequency domain representations of the waveforms and response functions. The functions used were the magnitude and phase functions of the associated spectra. This method was used because of the computational economy it afforded over multiple convolutions and because of the ease it afforded in specifying the distortion produced by the channel filter. Since all the components of this system were time-invariant, the signal computations were required only once in order to compute the pulse transmission characteristic, (the response of the entire linear system to excitation by a single signaling pulse). The response of this linear system to a sequence of signal pulses could then be computed as a linear combination of pulse transmission characteristics, the coefficients of the combination being the elements of the message sequence in (-1, +1) format. The inverse Fourier transform operation was carried out by numerical integration and produced the value of the signal component of $Z(t)$ at the sampling instant. The noise component of $Z(t)$, $Z_2(t)$, was produced by digital convolution of pseudo-random Gaussian samples with samples of the impulse response of the matched filter. After the threshold decision operation, the output binits were compared with the pseudo-random modulating sequence to determine the kind and number of errors which occurred.

The experiment which was performed was restricted to a study of linear delay distortion only. As a measure of severity of the channel distortion, the ratio of envelope delay at the edge of the signaling band (D) to the signaling baud length (T) was used. Thus D/T = 0 indicates no phase distortion, and D/T = 1 indicates that the envelope delay at one end of the signal bandwidth will be equal to +T, while the envelope delay at the other end will be equal to -T relative to the arrival time of the spectrum component at the center frequency.

On the following pages are graphical presentations of the results obtained from this computer simulation. Figure 3 is a plot of the magnitude of the signal spectrum used with this system -- the raised cosine spectrum, given by

$$|X(f)| = \begin{cases} \cos(2\pi f) & , \quad -\frac{1}{4} < f < \frac{1}{4} \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Also shown on this figure is the magnitude of the spectrum of the pulse transmission characteristic (matched filter output signal before sampling). This, of course, is given by

$$|Z(f)| = \begin{cases} [\cos(2\pi f)]^2 & , \quad -\frac{1}{4} < f < \frac{1}{4} \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Figure 4 shows the pulse transmission characteristic for the case of no distortion and for the case of linear delay distortion in the channel filter of magnitudes D/T = 1, 2, and 3.

Figure 5 contains a plot of the degradation in probability of error which occurs as the amount of linear delay distortion introduced by the channel filter is increased. The signal-to-noise ratio (ratio of signal pulse energy to noise spectral density) was set so that the probability of error would be .1 with no distortion provided by the channel filter. Also in Figure 5 is a plot of the probability of error versus distortion severity (D/T) where correction filters designed to compensate for the phase distortion have been introduced at the receiver. The compensating filters consisted of two and five cascaded all-pass networks whose center frequency and "bandwidth" (distance from $j\omega$ axis to the pole or zero) were adjusted to meet two criteria. These two criteria were:

1. The second derivative of the phase response function at f_o must be equal to the negative of the second derivative of the phase response function of the channel filter.
2. The third derivative of the phase response function at f_o must be zero.

Letting $\psi_c(f)$ be the phase response function of the correction network and $\psi_H(f)$ be the phase response function of the channel filter, the first criterion can be written

$$\left.\frac{\partial^2 \psi_c(f)}{\partial f^2}\right|_{f=f_o} = - \frac{\partial^2 \psi_H(f)}{\partial f^2}$$

and the second criterion is

$$\left.\frac{\partial^3 \psi_c(f)}{\partial f^3}\right|_{f=f_o} = 0$$

These requirements insure that for frequencies near f_o, the phase function of the correction filter will appear to be a quadratic function whose curvature will exactly cancel the curvature of the quadratic curvature of channel filter.

Figure 6 is a plot of the same information as in Figure 5, only the degradation in performance is now plotted as an equivalent decrease in signal-to-noise ratio for the system. For example, this graph shows that for linear delay distortion of magnitude D/T = 2, with no correction filter, the degradation in error performance is equivalent to a 3db drop in signal-to-noise ratio. Figure 7 is a plot of the sensitivity of the system error probability to mismatch between the channel distortion magnitude and the value of distortion magnitude for which the correction is set. For this plot, the channel distortion magnitude was held constant at D/T = 2 and the correction filter was changed to compensate for distortion magnitudes ranging from one to four.

Figure 8 indicates the effect of the use of different numbers of all-pass networks in the correction filters. Here, from one to five all-pass networks were used to provide the distortion correction and the improvement in error performance with the addition of each new network can be seen. Figure 9 is a comparison of the ideal pulse transmission characteristic for the system investigated (D/T = 0), the transmission characteristic which results from delay distortion of magnitude D/T = 3, and the same distorted characteristic after passing through a correction filter consisting of five all-pass networks. The improvement provided by the correction filter can be seen both in the increase in the value of the characteristic at the t = 0 sampling instant and in the reduction in the values of the characteristic at the other sampling instants (t = 4, 8, 12, etc.).

Graph 10 shows the phase characteristics found in the system when channel distortion of magnitude D/T = 2 is being corrected by a five all-pass network correction filter. It can be seen that the phase characteristic of the corrected pulse transmission characteristic is much more linear than that of the signal at the input of the correction filter. This indicates a reduction in the distortion of the transmission characteristic.

The results of this investigation have shown that in systems subject to linear delay distortion, it is possible to achieve significant reduction in system probability of error with the use of physical correction networks of reasonable complexity. It should be pointed out however, that the channel model used for this investigation was assumed to be time-invariant. In many communication systems of interest, however, the channel characteristics are time-variant. Also, other forms of distortion other than linear delay distortion are usually present, although it has been conjectured that in certain cases linear delay distortion is most significant in terms of probability of error effect[1]. In the future, it would be of interest to determine if the ideas investigated in this report could be employed in an adaptive arrangement for use with channels which are time-variant.

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[1] Sunde, E. D., Digital Tropascatter Transmission and Modulation Theory: B.S.T.J., Vol. 43, pp. 143-214, January, 1964 (Part 1).

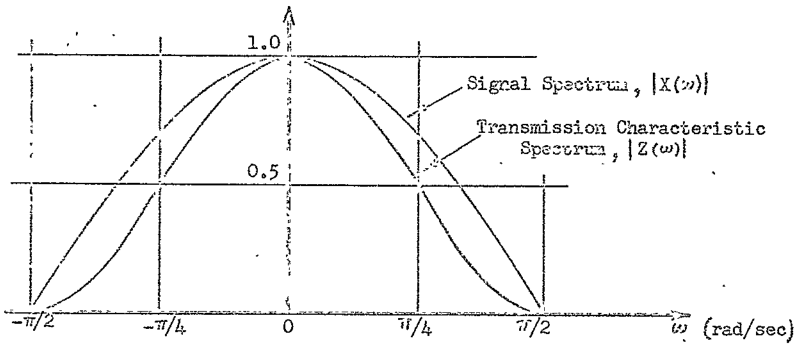

Figure 3 - Signal and Pulse Transmission Characteristic Spectra

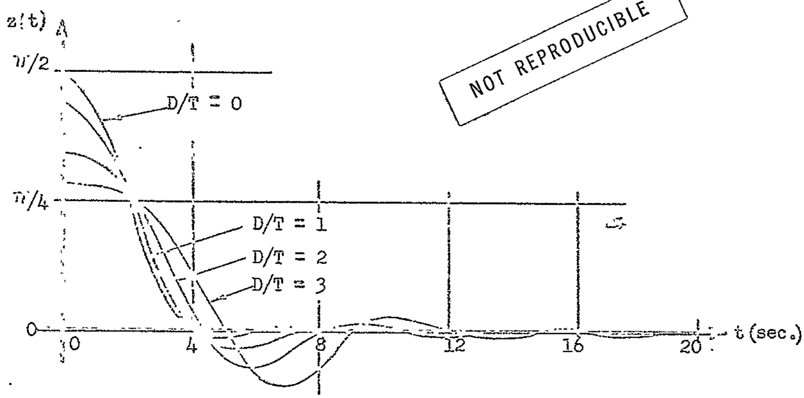

Figure 4 - Pulse Transmission Characteristics for Various Phase Distortions

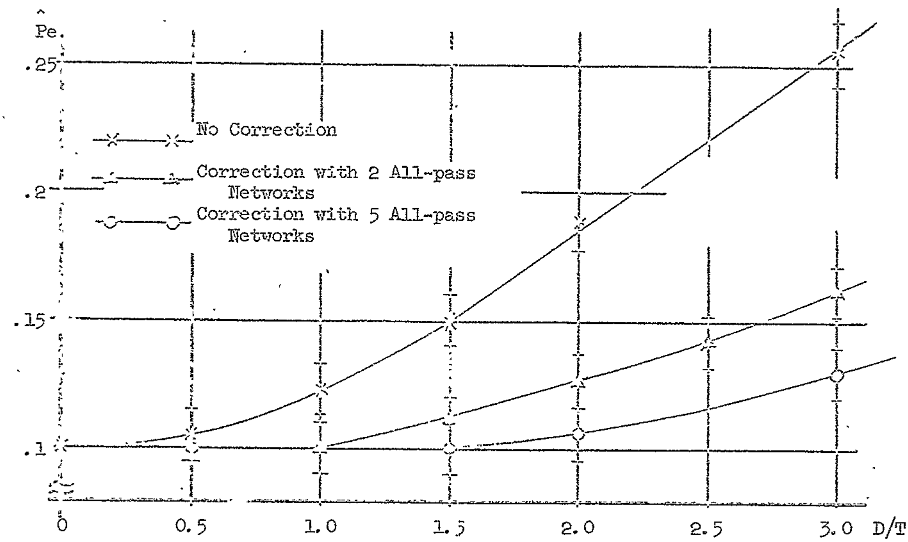

Figure 5 - Probability of Error Degradation versus Linear Delay Distortion

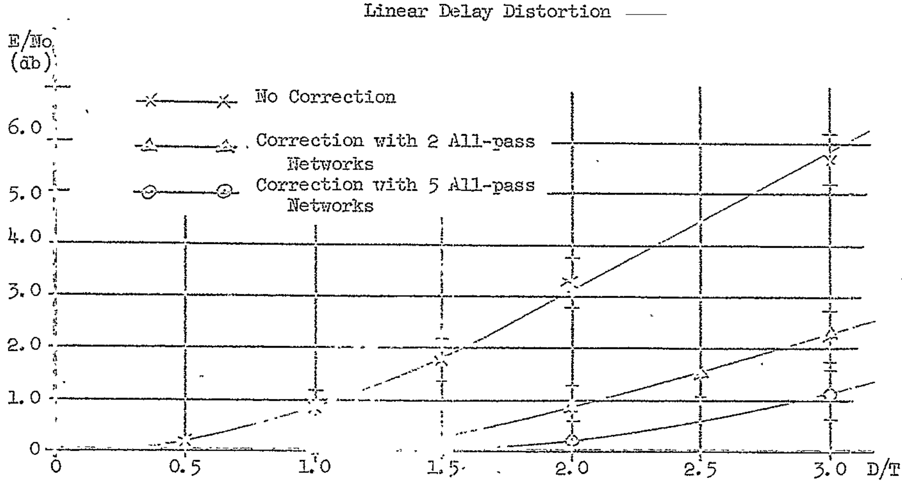

Figure 6 - Equivalent Signal-to-Noise Ratio Degradation versus Linear Delay Distortion

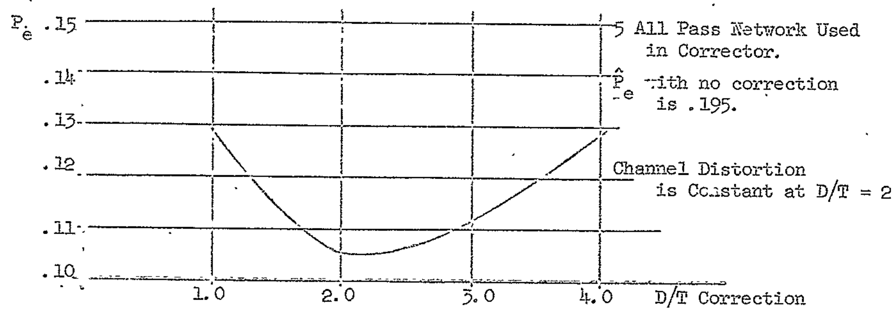

Figure 7-Sensitivity of All-Pass Correction to Channel - Corrector Mismatch

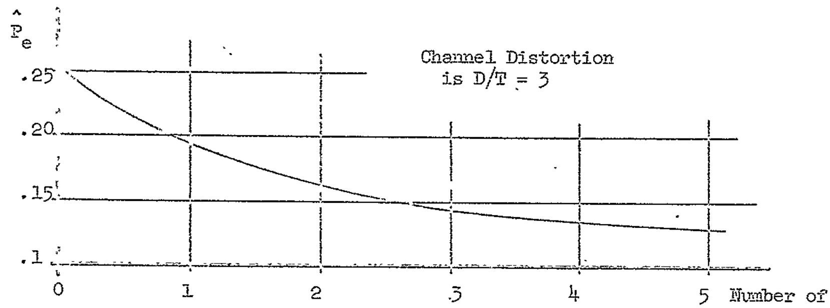

Figure 8 - Probability of Error versus Number of All-pass Networks in Correction Filter

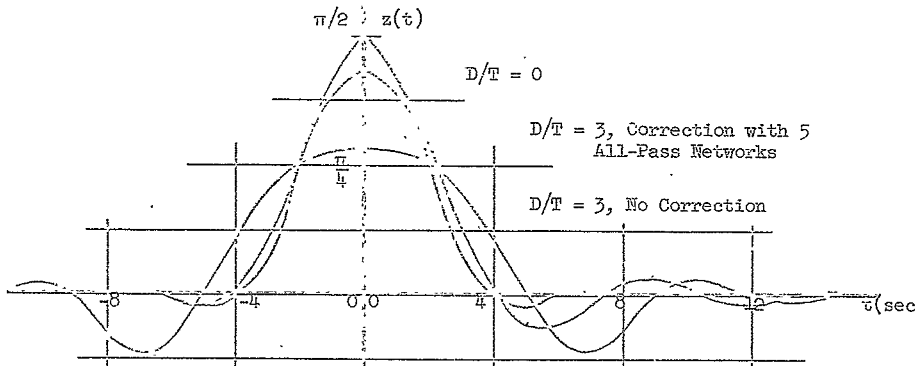

Figure 9 - "Corrected" and Uncorrected Pulse Transmission Characteristics